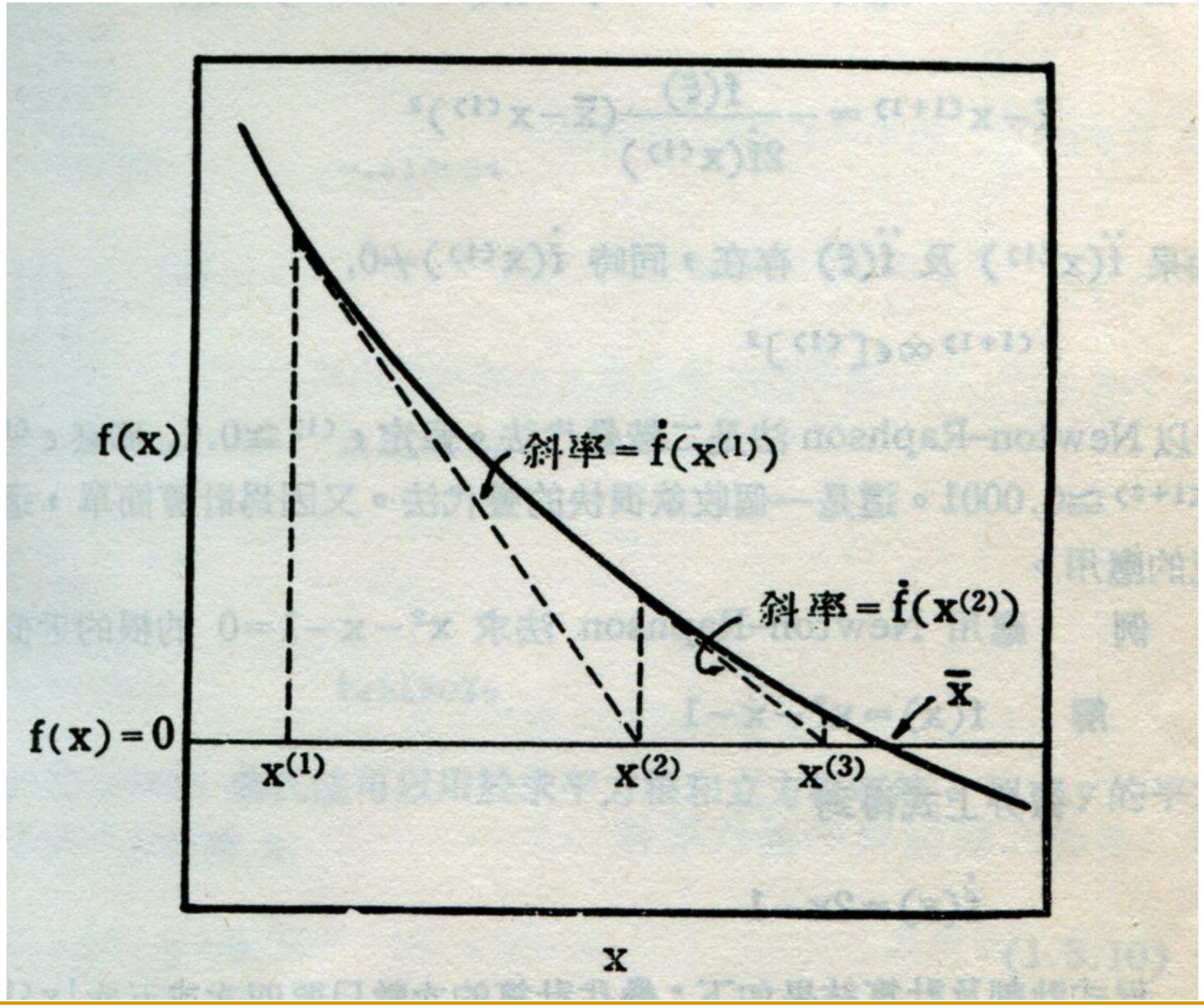
Programming Test Problem #1
Method of Newton-Raphson

Test Date: December 12, 2014


f(x)
斜率 = ḟ(x(1))
斜率 = ḟ(x(2))
x̄
f(x) = 0
x(1)
x(2)
x(3)
x

Iterative Solution

$$f(x) = 0$$

$$\text{If } x^{(1)} \to \bar{x}, \Rightarrow f(x) = f\left(x^{(1)}\right) + \left(x - x^{(1)}\right) f'\left(x^{(1)}\right) + \cdots$$

$$\therefore f(x) \cong f\left(x^{(1)}\right) + \left(x - x^{(1)}\right) f'\left(x^{(1)}\right)$$

$$\Rightarrow x^{(2)} = x^{(1)} - \frac{f\left(x^{(1)}\right)}{f'\left(x^{(1)}\right)}$$

$$\Rightarrow x^{(3)} = x^{(2)} - \frac{f\left(x^{(2)}\right)}{f'\left(x^{(2)}\right)}$$

$$\Rightarrow \cdots$$

$$\Rightarrow x^{(i+1)} = x^{(i)} - \frac{f\left(x^{(i)}\right)}{f'\left(x^{(i)}\right)}; \text{ and } i = 1, 2, 3, \cdots$$

Termination Criterion

$$\left| x^{(i+1)} - x^{(i)} \right| < \varepsilon$$

Say, $\varepsilon = 10^{-6}$

Questions to be Answered

$$f_1(x) = x^{2.3} - 6.37 = 0$$

$$f_2(x) = \sin x - 0.5x = 0$$

Programming Test Problem #2

Lagrange Interpolation

Test Date: December 12, 2014

Given Data

x	y
0.1	1.20998
0.3	1.68847
0.45	2.09434
0.62	2.59304
0.75	2.96104
0.86	3.33271
0.91	3.48612
0.93	3.54692
1.15	4.17276
1.30	4.51711

Tasks to be done

- Determine the values of y with *Lagrange interpolation* when x equals 0.2, 0.4, 0.6, 0.8, 1.0 and 1.2 respectively.
- Compare the interpolated values with the corresponding exact values obtained according to

$$y = e^{x}\left(\cos x + \sin x\right)$$

- A FORTRAN program is attached.

```
      PROGRAM LAGRANG (INPUT , OUTPUT )
C
C USING LAGRANGIAN INTERPOLATION FORMULA
      REAL L(10)
      DIMENSION X(10),Y(10)
      DATA X / 0.1 , 0.3 , 0.45 , 0.62 , 0.75 ,
     +         0.86 , 0.91, 0.93 , 1.15 , 1.30 /
      DATA Y / 1.20998 , 1.68847,2.09434,2.59304,2.96104,
     +         3.33271 , 3.48612,3.54692,4.17276,4.51711 /
      PRINT 10
   10 FORMAT(7X,≠X≠,9X,≠PHI(X)≠,4X,≠EXACT Y≠)
      N=10
      DO 20 XX = 0.2, 1.25 , 0.2
         PHI = 0.0
         DO 30 K = 1,N
C
            PRO = 1.0
            DO 40 I = 1,N
               IF (I.NE.K) THEN
                  PRO = PRO * ( XX - X(I) ) / ( X(K)-X(I))
               ENDIF
   40       CONTINUE
            PHI = PHI + PRO * Y(K)
   30    CONTINUE
C COMPARE THE RESULT (PHI) WITH THE EXACT VALUE
C
         EXACT = EXP(XX)*(COS(XX) + SIN(XX))
         PRINT ↑(4X,3F10.5)↑, XX,PHI,EXACT
   20 CONTINUE
      END
```

X	PHI(X)	EXACT Y
.20000	2.90039	1.43971
.40000	1.84721	1.95501
.60000	2.55064	2.53271
.80000	3.13414	3.14705
1.00000	3.77633	3.75605
1.20000	4.07466	4.29755